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FEATURES OF THE TRANSIENT PROCESSES IN A GROUND IN WHICH SOLAR ENERGY IS ACCUMULATED

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Algorithms of the transient processes arising in a ground in which solar energy is accumulated with the use of individual vertical heat exchangers when the work of solar collectors is stopped or resumed are proposed. The results of solving problems on the nonstationary heat conduction in semibounded bodies by the method proposed were compared with classical results.

Introduction. The accumulation of solar energy in a ground in a mild season for the purpose of its use in a cold season for municipal heat supply is a promising direction of nontraditional power engineering. The dynamics of accumulation of heat in a ground and its extraction with the use of an intermediate heat-transfer agent (water as a rule) heated by solar collectors or cooled in thermocompressors was simulated in [1–3]. In the work of solar collectors, there inevitably arise interruptions, which leads to a sharp change in the conditions under which the thermal processes proceed in a ground. These features were taken into account in the first approximation in the above-indicated investigations. In the present work, the transient thermal processes in a ground arising as a result of termination and resumption of the work of solar collectors are analyzed in more detail.

Termination of Heat Accumulation. At the end of a light day, the intensity of solar radiation decreases and a pump pumping water through a system of ground heat exchangers turns off. The pump generates a head expended for the formation of a heat-transfer-agent flow and for overcoming of the hydraulic resistance of a closed circulation loop. When the pump turns off, there arise damped oscillations of the head and of the heat-transfer-agent flow initiated by it. The frequency of change in the velocity of the oscillating liquid is determined by the expression

$$\omega_{\text{hyd}} \approx \sqrt{\frac{g}{H}}\,, \tag{1}$$

where g = 9.81 m/sec^2 and H is the head formed by the pump. The quantity ω_{hyd} is closely related to the frequency of oscillations ω_{h} of the heat-flux density q_0 determined by the Newton formula

$$q_0\,(t) = \alpha\,(t)\,(T_{\text{w}} - T_{\text{wall}})\,\frac{R_{\text{wall}}}{R_0}\,, \tag{2}$$

and, since $\alpha(t)$ is determined by the modulus of the velocity of the heat-transfer agent in a heat exchanger, the frequency of change in $q_0(t)$ and the period of temperature oscillations τ_{h} will be equal to

$$\omega_{\text{h}} \approx 2\omega_{\text{hyd}}\,, \quad \tau_{\text{h}} = \frac{2\pi}{\omega_{\text{h}}}\,. \tag{3}$$

The arising thermal waves attenuate on passage through a medium with a thermal resistance. The distance at which the temperature oscillations are decreased by e = 2.71 times is determined from the formula [4]

$$\frac{l}{2\pi} = \sqrt{\frac{a_{\text{m}}\tau_{\text{h}}}{\pi}}\,. \tag{4}$$

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For for ground masses with $a_m \sim 10^{-6}$ m^2/sec and a circulation loop of length ~300 m, the value of the right side of (4) is $0.88 \cdot 10^{-3}$ m ($\tau_h \sim 2.5$ sec) at a pump head $H \sim 0.02 \cdot 300 = 6$ m. Thus, the change to the value of $q_0 = 0$ (end of the damped wave process) will lead to a marked transformation of the temperature profile in a ground at a small distance from the wall of a heat exchanger.

The temperature distribution in a ground mass in which heat is accumulated is defined by the dependences [1–3]

$$\frac{T - T_m}{T_0 - T_m} = \begin{cases} (1-\eta)^3 (1 + 3\eta - A_m\eta), & A_m \le 4; \\ (1-\eta)^{A_m}, & A_m > 4, \end{cases}$$

$$A_m = \frac{q_0 (R - R_0)}{\lambda_m (T_0 - T_m)}; \quad \eta = \frac{r - R_0}{R - R_0}. \tag{5}$$

The first formula (5) is transformed, at $q_0 = 0$ ($A_m = 0$), into

$$\frac{T - T_m}{T_0 - T_m} = (1-\eta)^3 (1 + 3\eta). \tag{6}$$

Because of the intensive damping of heat perturbations, the temperature profile corresponding to the change from the "pumping" of heat ($q_0 > 0$) to its natural drift ($q_0 = 0$) cannot substantially differ in structure from profiles (5). Formula (6) can be used in the approximate form

$$\frac{T - T_m}{T_0 - T_m} = (1-\eta)^B (1 + B\eta), \tag{7}$$

which satisfies the condition $q_0 = 0$ at any finite value of the parameter B. In the process of heat accumulation, $A_m >> 10$. If $B = A_m$, the difference between the fullnesses of profiles (7) and the initial profile (5) is equal to

$$\Delta = B\eta (1-\eta)^B. \tag{8}$$

Investigation of (8) to find an extremum gives

$$\eta_{ext} = \frac{1}{1+B}. \tag{9}$$

The second derivative of (8) with respect to η is negative at $\eta = \eta_{ext}$; therefore, $\Delta(\eta_{ext}) = \Delta_{ext} = \Delta_{max}$. As follows from (9), $\eta_{ext} < 0.1$ at $B \ge 10$, which corresponds to the above-described notions of the zone where the temperature profile is mainly transformed. We will calculate the change from the accumulation of heat to its drift in the following order. Profile (5) is transformed into profile (7) during several periods of damped oscillations of the quantity q_0. The value of $B(t_{d,b})$ is calculated by the integral condition of accumulated-energy conservation at F = idem ($F = T_0, R$).

Natural Drift of Heat. At $t > t_{d,b}$, profile (7) is diffusively transformed, with the result that $B(t)$ changes from $B(t_{d,b})$ to $B(t_{d,e})$. The limiting minimum value of the parameter B is equal to $B_{min} = 3$ (see formulas (6) and (7)). The dynamics of the determination process is dictated by the fundamental heat-conduction equation and the integral energy equation. Using methods described in [1–3], we obtain

$$\frac{dT_0}{dt} = -a_m \frac{T_0 - T_m}{R^2} B (B+1), \quad \frac{dR}{dt} = 6 \frac{a_m}{R},$$

$$Ei = 2\pi\rho_m c_m (T_0 - T_m) \frac{R^2}{(B+2)(B+3)} \left[3Z + \frac{16Z}{(B+4)} \right] = \text{const}. \tag{10}$$

Expressions (10) were obtained assuming that $R >> R_0$, which represents the facts. The system of equations (10) is closed and the changes in $T_0(t)$, $R(t)$, and $B(t)$ during the natural drift of heat can be determined by the initial conditions at the instant of time $t_{d,b}$.

Change from Drift to Accumulation. This stage begins when a pump turns on and a flow of an intermediate heat-transfer agent arises. Since the heat-transfer agent–ground massive system cannot be in complete heat equilibrium, α_0 and, accordingly, q_0 increase sharply when the heat-transfer agent begins to move. As the heat-transfer agent, cooled in the process of drift, is displaced by a heat-transfer agent heated by solar collectors, the regime of operation of a setup approaches the calculated one. The time of establishment of the calculated regime τ_p can be determined in the first approximation on the assumption that the heat-transfer agent moves in a tube system in a piston regime:

$$\tau_p = \frac{L}{v_w}, \quad (11)$$

where L is the length of the contact surface of a heat exchanger with the ground and v_w is the velocity of travel of the heat-transfer agent. Thus, it is assumed that (a) profile (7) at $B = B(t_{d,e})$ is transformed rapidly into profile (5) at F = idem ($F = Ei$, T_0, R) and (b) for the time τ_p, the parameters of the heat-transfer agent become equal to the calculated ones determined by the work of solar collectors. The indications of the establishment of this regime are definite values of q_0 or T_0 attained for the time τ_p. The first parameter is determined more easily than the second one when the characteristics of solar collectors and the intensity of solar radiation are known. Subproblem (b) is solved within the framework of the complete system of equations (1)–(3) at the initial conditions determined by the solution of subproblem (a).

This is an exhaustive description of the transient processes. Within the framework of the above-described notions we have developed software for solving problems on the transient processes determining the change from the accumulation of heat to its natural drift and, conversely, from the heat drift to the "pumping" of energy.

Analysis of Solutions. The above-described problems can be solved only by a numerical method. Therefore, we will consider their features using concrete examples. Let heat be accumulated in a ground ($\rho_m = 1.84 \cdot 10^3$ kg/m^3, $c_m = 1.15 \cdot 10^3$ J/(kg·K), $\lambda_m = 1.42$ W/(m·K), $T_m = 10^oC$) with the use of a vertical coaxial heat exchanger (R_0 = 0.054 m, R_{wall} = 0.050 m, R_{in} = 0.040 m, Z = 50 m, λ_{wall} = 17.5 W/(m·K)), in which water heated by solar collectors ($T_w < 60^oC$) circulates (G_w = 5.0 kg/sec). To ensure that the changes in the functions studied were clearly defined, the initial density of a heat flow is assumed to be high (q_0 = 1000 W/m^2). In the case where heat is accumulated by an individual heat exchanger, the radius of heat propagation R and the parameter A_m increase rapidly. As was shown in [1–3], at $A_m \sim 300$ in typical grounds, the thermal head formed $\Delta T = (T_w - T_m) \sim 40$ K is depleted practically entirely in the neighborhood of $R \sim 1$ m adjacent to the heat exchanger. Therefore, the further pumping of heat into the ground leads to an indefinitely large increase in the accumulation region ($R \to \infty$) and to the accumulation of energy in it; the potential of this energy is equal to the initial potential of the ground T_m. Because of this, it is necessary to control the work of an individual heat exchanger, e.g., by restriction of the heat-propagation radius, which is taken, in the example considered, to be equal to R_e = 5.0 m.

The changes in the main parameters during the first eight hours (interval of work of a solar collector) of heat accumulation are presented in Fig. 1a. It is seen that $R(t)$ and $A_m(t)$ increase very rapidly in the initial period. After t_e = 1.47 h from the beginning of work, these quantities reached the values R = 5.006 and A_m = 111.4. The temperature T_0 changed from 10 to 40.84^{o}C and the temperature T_w changed from 12.23 to 41.67^{o}C. When R = 5.006 m was attained, heat was pumped in the regime $R = R_e$ = const and the density of the heat flow q_0 was decreased, which caused a decrease in A_m. The temperature T_0 increased insignificantly for 0.5 h after t_e and then decreased. The finite values of the parameters were as follows at the end of the work of the solar collectors (8 h): T_w = 21.35^{o}C, T_{wall} = 21.34^{o}C, T_0 = 21.31^{o}C, A_m = 30.19, q_0 = 97.97 W/m^2, and $Ei = 0.251 \cdot 10^9$ J. The change in T_0 becomes more smooth when the initial value of q_0 decreases.

When the accumulation of heat is changed to its natural drift, the initial value of B = 45.196. The graphs of change in the main functions $B(t)$, $T_0(t)$, and $R(t)$ during the remaining 16 h before the beginning of the next period of work of solar collectors are given in Fig. 6. The finite parameters of the drift are as follows: T_0 = 12.77^{o}C, B = 21.46, and R = 5.061 m. The radius of heat propagation was increased only by 0.055 m. However, the number of such noncontrolled drifts will be 180 in the process of heat accumulation plus 180 drifts in the process of discharge. It should be noted that B did not reach the limiting value B_{min} = 3 for the period of heat drift.

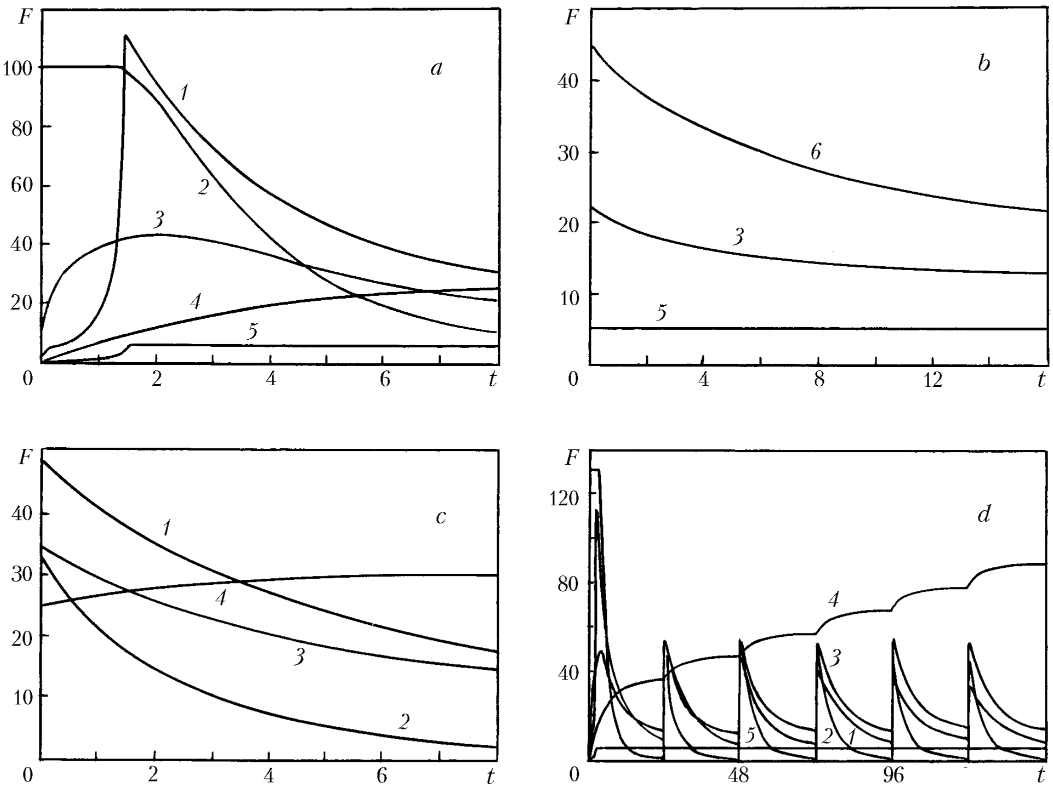

Fig. 1. Change in the parameters F ($F = A$, q_0, T_0, Ei, R, B) in the process of heat accumulation (a), natural heat drift (b), resumption of heat accumulation ($q_0(t_{e.d}) = 322.5$ W/m^2) (c), and continuous heat accumulation (d): 1) A; 2) q_0; 3) T_0; 4) Ei; 5) R; 6) B. q_0, 10 W/m^2; T_0, ^{o}C; Ei, 10^7 J; R, m.

The resumption of heat pumping transformed the temperature distribution (7) into distribution (5) with $A_m = 13.324$. During $\tau_p = 29$ sec (see (11)), the value of $R = 5.061$ m remained practically unchanged and, at $q_0 = 322.5$ W/m^2, solution of problem (b) led to the following results: $T_0 = 33.76^o$C and $A_m = 47.86$. The subsequent pumping of heat for 8 h at $R = 5.061$ m = const is illustrated by the graph of change in the main functions (see Fig. 1c).

The above-described algorithm can be easily extended to the whole accumulation season. Control of energy accumulation allows one to substantially decrease the radius of heat propagation. Despite the fact that the radius R increases by a comparatively small value for 16 h of one drift ($\Delta R \sim 0.1$ m), this radius will increase by a large value ($\Delta R \sim 10$–20 m) during an accumulation season including 180 interruptions. To prevent an increase in the heat region, it is necessary to control the pumping of energy for the 24 h in a day. An example of such control performed with the aim of limiting $R = 5.0$ m is presented in Fig. 1d. To realize a continuous heat accumulation, it is necessary to use twenty-four-hour accumulators. Analysis of the regime considered has shown that 80–85% of the twenty-four hours' energy is pumped for the time of active work of the solar collectors (see Fig. 1d). The results of calculations point to the fact that the volume of a twenty-four-hour water accumulator with a temperature 75^{o}C can be about 0.167 m^3/kW.

Comparison of the Integral Method with a Classical Method. We will supplement the comparison of the solutions of the problems on the nonstationary heat conduction of spatially nonbounded bodies by classical methods and the above-described integral method, presented in [1], with two known problems on heating of a semibounded body [5]:

(a) with a graduated increase in the temperature of the face T_m to T_0 and subsequent maintenance of it at this level;

(b) in the case of sudden heat supply to the face and q_0 = const.

When problem (a) is solved, both approaches give identical formulas:

$$q_0(t, 0) = \frac{\lambda_m (T_0 - T_m)}{\sqrt{k a_m t}} , \quad (12)$$

where $k = \pi$ (classical solution) and $k = 3.33$ (integral method). The temperature distributions $T(t, x)$ are practically identical, except for the temperature distributions in the region $\eta = x/X > 0.6$, where the classical solution should give a somewhat fuller profile.

The solution of problem (b), obtained by the integral method at $A_m > 4$,

$$\frac{dT_0}{dt} = \frac{a_m (T_0 - T_m)}{q_0^2 t^2} \left[\rho_m c_i (T_0 - T_m) - \frac{q_0^2 t}{\lambda_m (T_0 - T_m)} \right]^2 , \quad (13)$$

is not reduced in all probability to quadratures. At $A_m >> 1$, expression (13) becomes simper and is reduced to the approximate expression

$$T_0 - T_m = \frac{q_0}{\lambda_m} \sqrt{a_m t} ; \quad (14)$$

the numerical coefficient on the right side of (14) is equal to unity, and in the case of the exact (classical) solution, to $2/\sqrt{\pi} = 1.128$ [5].

Conclusions. More exact algorithms (as compared to the algorithms presented in [1–3]) of the transient processes arising in a ground in which solar energy is accumulated as a result of the termination and resumption of the work of solar collectors have been developed. The number of problems on nonstationary heat conduction in unbounded media which can be solved analytically is very small. Therefore, estimation of the accuracy of determining the thermophysical parameters (λ, ρ, c) of actual ground masses allows the conclusion that the method proposed can be considered as wholly satisfactory.

NOTATION

A, parameter; a, thermal diffusivity, m^2/sec; c, specific heat capacity, J/(kg·K); Ei, energy, J; G, flow rate of an intermediate heat-transfer agent, kg/sec; g, free fall acceleration, m/sec^2; H, head formed by a pump, m; k, coefficient; L, length of the contact of a heat-exchange surface with a ground, m; l, length of a thermal wave, m; q, density of a heat flow, W/m^2; R, radius of the heat propagation, m; R_0, outer radius of a heat-exchanger tube, m; r, x, coordinates, m; T, temperature, ^{o}C; t, times, sec; v, velocity of a flow, m/sec; Z, working height of a heat exchanger, m; X, characteristic length, m; α, heat-transfer coefficient, W/(m^2·K); Δ, difference between values; η, dimensionless coordinate; λ, heat-conductivity coefficient, W/(m·K); ρ, density, kg/m^3; τ, interval, sec; ω, frequency, rad/sec. Subscripts: w, water; in, inner; hyd, hydraulic; d, drift; e, end; m, mass; b, beginning; p, piston; wall, wall; h, heat; ext, extremum; max, maximum; min, minimum.

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